Bearing Capacity of Strip Footings in Regions of Medium Seismicity: Reappraisal of the Pseudostatic Approach in Code-based Design in Light of Recent Computational Results

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ABSTRACT
A considerable number of analytical and numerical solutions have been presented hitherto to evaluate the seismic bearing capacity of shallow foundations, including limit equilibrium methods, limit analysis, method of characteristics, and finite element analyses. Inertia forces applied on the foundation and in the soil mass reduce the static bearing capacity. Since static design is mostly controlled by settlement, the effective safety factor is usually much higher, providing justification for the adoption of pseudostatic approaches. This applies in particular to medium seismicity environments, where a smooth transition from the static to the seismic case is essential in order to avoid overdesign. The paper first compares closed-form solutions for inclined and/or eccentric loading in terms of the resulting dome-shaped bounding surfaces. Recent findings from computational studies are summarized next. A widely available FEM code incorporating a Mohr-Coulomb yield criterion is applied to re-calculate typical situations and assess the suitability of numerical analyses in routine design. Finally, a proposal for code provisions is presented.

1 INTRODUCTION
In most cases of routine building design, dimensioning of the foundation is governed by serviceability aspects while bearing capacity is of secondary importance. On the other hand, due to the non-linear nature of soil behaviour, knowledge of the maximum bearing load of a foundation provides information on the utilization degree of the soil strength that can be used for estimates of the effective stiffness in serviceability analysis.

The complete analysis of the response to seismic loading would require consideration of both kinematic and inertial response, a procedure that is extremely complex and, due to the absence of severe failures apart from those attributed to liquefaction, also economically not justified. Hence, simplified approaches are usually employed. They represent the seismic effects by pseudostatic accelerations that induce i) eccentric and inclined inertial forces as imposed by the superstructure and ii) acceleration forces in the soil mass inertia. For the application in design, partial safety factors for actions and resistances are provided by the pertinent building codes.

The static solution for the basic case of a shallow strip footing has been subject of an immense number of studies since the 1920s. Various methods have been employed: limit equilibrium methods, limit analysis, the method of characteristics (or slip line method), and conventional numerical methods. The latter are becoming increasingly popular due to their versatility in capturing effects of irregular foundation geometry and soil stratigraphy. They require, however, particular attention to modelling details. Methods for the seismic response using the pseudostatic approach follow the same principles.

For the static case, the ultimate bearing load is customarily expressed as a linear combination of three bearing capacity components accounting for the effects of cohesion, lateral surcharge, and soil weight. The respective bearing capacity factors $N_c$, $N_q$ and $N_y$ depend solely on the friction angle of the soil. The shape of the foundation, the load inclination and eccentricity, and possibly the embedment depth are considered by means of multiplicative correction factors, Das (2009). A typical example is the equation given in the European Standard for Geotechnical Design, EN 1997-1 (Eurocode 7), that is based on a former version of the German Standard DIN 4017 on bearing capacity calculation for foundations. To incorporate inertial effects, a modification of the basic equation using reduction factors to the static bearing capacity factors, yielding their seismic counterparts $N_{c,E}$, $N_{q,E}$ and $N_{y,E}$, is often adopted in practice.

Another alternative consists in defining 3D bearing surfaces by means of a single equation comprising combinations of vertical force, horizontal force and moment that induce bearing failure of the foundation. Incorporation of inertial effects is accomplished by altering the static equations, often in a less comprehensive way for the user compared to the classical three-term equation. A typical representative of this type of design equation is that given in Eurocode EN 1998-5 (Eurocode 8, Part 5). Similar bearing surfaces may also be constructed using the three-term capacity equation.

A detailed overview of the methods proposed hitherto to calculate the seismic bearing capacity coefficients together with the assumptions made are given by Cascone & Casablanca (2016) and Pane et al. (2016). The findings of these and other recent studies together with our own investigations are used in the sequel to assess the accuracy and limits of the two calculation methods described above.

2 PSEUDOSTATIC THREE-TERM EQUATION
2.1 General equation
The adaption of the classical bearing capacity equation to include structure and soil mass inertia forces reads:
q_{ult} = c N_c i_c E + q N_q i_q E + \frac{1}{2} V B N_s i_s E \quad [1]

where c is the cohesion, q the lateral surcharge, v the unit weight of the soil beneath the footing, B the footing width, N_c, N_q and N_s bearing capacity factors depending on the friction angle $\phi$ of the soil, $s_c$, $s_q$ and $s_s$ foundation shape factors, and $i_c,E$, $i_q,E$ and $i_s,E$ seismic inclination factors. The latter are obtained by multiplying the static inclination factors by appropriate reduction factors $e_{c,E}$, $e_{q,E}$ and $e_{s,E}$.

\[
i_{c,E} = i_c E, \quad i_{q,E} = i_q E, \quad i_{s,E} = i_s E \quad [2]
\]

Use of Eq. 1 implies that soil is a cohesive-frictional material with linear Mohr-Coulomb failure envelope, and cohesion and friction mobilized at the same shear strain $\gamma$.

2.2 Bearing capacity factors

The bearing capacity factors $N_q$ and $N_c$ are known exactly,

\[
N_q = \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)e^{\tan \phi}, \quad N_c = (N_q - 1)\cot \phi \quad [3]
\]

whereas for $N_s$ several solutions have been proposed as summarized by Magnan et al. (2003), or Orr (2010). The EN 1997-1 equation derives from that given in DIN 4017

\[
N_s = 2(N_q - 1)\tan \phi \quad [4]
\]

It yields a higher bearing load than all other equations in the literature, and has been introduced to reflect the experimental findings on large scale tests on footings resting on uniform sand conducted by the Degebo in Berlin in the 1960s. These tests are well documented in a series of extensive reports including investigations on the effects of embedment, load inclination and eccentricity, as well as on the foundation shape. For an overview refer to Muhs & Weiss (1970), (1973).

In the last two decades, numerical results have been obtained using the method of characteristics (slip line method), Martin (2005), Cascone & Casablanca (2016), or by explicit numerical solution of the simultaneous partial differential equations, Smith (2005). Meanwhile, these solutions are considered as exact. It should be mentioned that the results strongly depend on the contact conditions at the interface footing-soil, expressed in terms of a global interface friction angle $\delta$. For a perfectly smooth interface ($\delta = 0$) the bearing capacity factor is approximately half the value of that for a perfectly rough interface ($\delta = \phi$). At $\phi = 30^\circ$, for example, $N_i = 7.65$ for $\delta = 0$, and $N_i = 14.75$ for $\delta = \phi$. The contact stress distribution and the shape of the failure mechanism depend on the interface condition. This has to be considered when such solutions are verified by conventional numerical methods (FEM, FDM). In the paper by Cascone & Casablanca (2016), an approximation for $N_i$ is given for the two limiting cases of the interface contact condition.

Another point that deserves attention is the selection of the friction angle. The Degebo tests were back-analysed for the friction angle at peak as determined by direct shear tests. The use of the residual or critical friction angle is not recommended as this would yield the same bearing capacity for loose and dense sand.

Investigations by Smith (2005) to assess the effects of superposition in the bearing capacity equation in cohesionless soil showed that coupling of the two effects (lateral surcharge and soil weight) results in an increase in bearing resistance. The ratio between the fully-coupled solution and the superimposed solution is denoted by $\mu$, and its value depends on the surcharge ratio $q/\gamma B$, on $\phi$, and on $\delta$. It is shown that $\mu \leq 1$ always holds. For example, for $\phi = 30^\circ$, $q/\gamma B = 1$, and for a rough footing, $\mu$ approximately equals 1.20. This fact provides an additional margin of safety when using the classical three-term equation in design.

2.3 Shape factors

The shape factors have been determined mostly from model and large scale tests, Magnan (2003). They are not considered further herein.

2.4 Load eccentricity

Load eccentricity is customarily considered by reducing the strip foundation width $B$ to an effective one $B'$

\[
B' = B(1 - 2e/B), \quad e = M/V \quad [5]
\]

with $M$ denoting the moment and $V$ the vertical force.

Eccentrically loaded footings on cohesionless soil have been investigated by Krabbenhoft et al. (2012) by lower bound calculations based on the finite element method. The simultaneous presence of soil weight and lateral surcharge has been considered. For vanishing lateral surcharge it was shown that eccentricity is well-captured by Eq. 5. For the coupled case (both $q$ and $\gamma$) the complete analysis for various values $q/\gamma B$ shows that the results when split into two additive terms reproduce approximately the (1-2e/B) assumption.

Eccentricity was addressed also by Paolucci & Pecker (1997). Applied to Eq. 1 their suggestion is slightly different, with $B' = B(1 - 2e/B)^{1/3}$.

2.5 Load inclination factors

The static inclination factors given in EN 1997-1 are dependent on $c$ and $\phi$ whereas in the current edition of DIN 4017 (2006) this dependency has been retained only for $i_c$. The respective DIN-equations for a strip footing are:

\[
i_c = \frac{i_c N_q - 1}{N_q - 1} \quad \text{for } \phi > 0 \quad [6a]
\]

\[
i_c = 0.5 + 0.5 \sqrt{1 - \frac{H}{Bc}} \quad \text{for } \phi = 0 \quad [6b]
\]

\[
i_q = \left(1 - \frac{H}{V}\right)^2 \quad [7]
\]
\[ i_y = \left(1 - \frac{H}{V}\right)^3 \]  

with H denoting the horizontal force on the foundation.

The above equations are compared with the numerical results by Cascone & Casablanca (2016) in Figure 1 showing very good agreement for \( i_q \) and \( i_y \) but considerable deviation for \( i_c \).

Inclination factors due to structure-inertia dependent on the seismic coefficient for \( \varphi = 30^\circ \). The data points are from Cascone & Casablanca (2016).

Incorporation of soil mass inertia yields a reduction of the inclination factors, cf. Eq. 2. Inertia effects are modelled by a seismic coefficient in the horizontal and vertical direction \( k_h \) and \( k_v \), respectively. The vertical component is often ignored, and this is also the case herein. Our proposal for the seismic reduction factors \( e_{c,E} \), \( e_{q,E} \) and \( e_{y,E} \) is synthesized from the suggestions by Paolucci & Pecker (1997) and the numerical results recently presented by Pane et al. (2016) and Cascone & Casablanca (2016). The latter contains a detailed parametric study using the method of characteristics and the finite element method.

The reduction factor for the cohesion term \( e_{c,E} \) depends only weakly on the soil inertia. Hence, we follow the suggestion by Cascone & Casablanca (2016) and Pane et al. (2016) to set

\[ e_{c,E} = 1 \]  

The other two factors are derived by curve fitting from the numerical values by Cascone & Casablanca (2016):

\[ e_{q,E} = (1 - k_h / \tan \varphi)^{0.3} \]  

\[ e_{y,E} = (1 - k_h / \tan \varphi)^{0.45} \]

It should be mentioned that \( e_{y,E} \) depends on the contact conditions at the interface footing-soil, the exponent 0.45 closely fitting rough contact that prevails in real-life situations. The quality of the approximation is visible in Figure 2. Eqs 10 and 11 are in good agreement with the findings by Pane et al. (2016) and Paolucci & Pecker (1997) that recommend the use a single equation for \( e_{q,E} \) and \( e_{y,E} \) with an exponent approximately equal to 1/3 in Eq. 10 and 11.

A further point worthy of investigation refers to the accuracy of the superposition of the structure-inertia with the soil-inertia effects by ignoring the cross-interaction effects as presumed by Eq. 2. Both studies by Cascone & Casablanca (2016) and Pane et al. (2016) indicate that such a superposition is - despite the non-linearity of the problem - a very good approximation with minor deviation on the unsafe side.

In this convenient representation, bearing strength surfaces are defined by combinations of vertical load \( V \), horizontal shear \( H \), and moment \( M \) acting on the foundation that cause bearing failure in the foundation soil. Gottardi & Butterfield (1994). All admissible sets of load are located within that surface

\[ f(H,V,M) \leq 0 \]

The respective equation is given in EN 1998-5, Annex F, for strip footings resting either i) on purely cohesive soil of undrained strength \( s_u \), or ii) on cohesionless soil of friction angle \( \varphi \). A lateral surcharge is not included in that solution. The principles of the method and the details of the derivation are outlined in Salencon & Pecker (1995) and Pecker (1997). The incorporation of partial safety factors in Eq. 12 created some confusion in the implementation, as ultimate limit state verifications for the static case are conducted by applying safety factors either to resistances or to material strength parameters. To remove ambiguity, we set here all partial safety factors equal to 1. Furthermore, we ignore for simplicity the vertical seismic action. The EN 1998-5 equation then reads

\[ \frac{(1 - eF)^{0.5} (\beta R)^{c_1}}{V^a (1-m r^k)^{k - \frac{c_1}{c}} - 1 \leq 0} \]

where
\[ \nabla = \frac{V}{V_{\text{max}}} \quad , \quad R = \frac{H}{V_{\text{max}}} \quad , \quad \Phi = \frac{M}{BV_{\text{max}}} \]  
\[ V_{\text{max}} = (\pi + 2)\alpha B \quad \text{for purely cohesive soil} \quad [15] \]
\[ V_{\text{max}} = \frac{1}{2}VB^2N_y \quad \text{for cohesionless soil} \quad [16] \]

The parameters \( a, b, c, d, e, f, m, k, k', c_t, c_M, c'_M, \beta, \gamma \) depend on the soil type as given in Table F.1 of EN 1998-5. \( \alpha \) is a dimensionless inertia force for the soil

\[ F = \frac{\alpha S y B}{s_u} \quad \text{for purely cohesive soil} \quad [17] \]
\[ F = \frac{\alpha S}{\tan \phi} \quad \text{for cohesionless soil} \quad [18] \]

where \( \alpha \) is the ratio of the design ground acceleration on type A ground to the acceleration of gravity \( g \), and \( S \) is the soil factor as specified by EN 1998-5. Hence, it is assumed that the seismic coefficient \( k_s \) in the soil equals \( \alpha S \), which is not always true.

4 COMPARISONS AND FEM ANALYSES

We first compare the 3D dome-shaped bearing surfaces defined by combinations \( H, V, \) and \( M \geq 0 \). For the DIN-based three-term equation, one first selects a pair of values \( H, V \) and using an iterative numerical solver finds the moment \( M \) for which the vertical bearing load of the footing equals \( V \). One proceeds correspondingly for the EN 1998-5 procedure, Eq. 13. Plots of 2D interaction curves on cross-sections through the dome are obtained accordingly.

The comparisons reported here are for \( q = 0 \), as the EN 1998-5 formula does not include the effects of \( q \). The dimensionless soil inertia force \( F \) in the plots is fixed: \( F = 0 \) and 0.5. Bear in mind that \( F \) includes also the shear strength of the soil. Figures 3 to 6 compare the shear bearing surfaces. Figures 7 to 10 depict the corresponding 2D interaction curves. It can be seen that both procedures yield comparable results.

Figure 3. Bearing surface for the DIN-based three-term equation for purely cohesive soil; \( F = 0 \).

Figure 4. Bearing surface for the DIN-based three-term equation for cohesionless soil; \( F = 0 \).

Figure 5. Bearing surface for the explicit yield design theory equation of EN 1998-5 for purely cohesive soil; \( F = 0 \).
In order to assess the accuracy of FEM codes used in design, calculations with the program PLAXIS 2D were carried out in the pseudostatic modus. The study by Cascone & Casablanca (2016) provided guidance for the implementation. The model under study consists of a strip footing of width $B = 2$ m modelled as plate of finite rigidity resting on a soil stratum with dimensions $30 \times 10$ m (width x depth). A variable mesh density with a finer mesh at the edges of the footing was selected according to the recommendation by Loukidis & Salgado (2009). An elasto-plastic constitutive model was assumed for the soil with Young’s modulus $E = 30$ MPa, Poisson’s ratio $\nu = 0.2$, and a linear Mohr-Coulomb failure envelope with an associated flow rule. For the calculation of the bearing capacity factors unit weight $\gamma$ in kN/m$^3$, cohesion $c$ in kPa, and lateral surcharge $q$ in kPa were switched on/off or assigned very small values: $c/\gamma = 0.01/0.01$ for computing $N_c$; $c/\gamma = 0.01/0.01/0.015$ for $N_c$, 0.0120/0 for $N_s$. The ultimate bearing load was determined by increasing the load whereby the load step increment is automatically adjusted by the numerical code. The analytical, exact values for $N_e$ and $N_s$ are recovered. For $N_u$, however, as outlined above, one has to adequately consider the contact conditions at the interface soil-foothling. For reproducing the perfectly smooth condition of the slip line method, a triangular normal stress distribution has been imposed. The results compare well ($N_u = 17.58$ at $\phi = 35^\circ$). Results for rough contact strongly depend on the rigidity ratio between the plate simulating the foundation and the underlying soil. A parametric study indicates that $N_u$ approaches the slip line method value ($N_u = 34.48$ at $\phi = 35^\circ$) for bending stiffness $E_I$ approximately equal to $2 \times 10^6$ kNm$^2$/m.

The pseudostatic case with soil inertia is modelled by imposing a horizontal volumetric force to the soil mass corresponding to the seismic coefficient $k_s$. The numerical results obtained are in close agreement with those by Cascone & Casablanca (2016).

We further investigated the effects of load eccentricity showing that the use of the effective width $B_e$ according to Eq. 5 is a very good approximation on the safe side: For $\phi = 35^\circ$ and $e = 0.5$ m we obtained $B_e = 0.5$ m from Eq. 5 and $B_e = 0.52$ m from the FEM analysis.
Figure 8. Comparison of the interaction curves for the DIN-based three-term equation and the EN 1998-5 equation for purely cohesive soil; $F = 0.5$.

Figure 9. Comparison of the interaction curves for the DIN-based three-term equation and the EN 1998-5 equation for cohesionless soil; $F = 0$. 
5 CONCLUSIONS

The dome-shaped bearing surface of V-H-M sets causing failure computed from the classical three-term equation is comparable to that derived from the yield design theory. Discrepancies due to different $N_F$ - values are eliminated through normalization to the ultimate vertical centric load.

A parametric study with a widely-used finite element code demonstrated its suitability to reproduce available exact solutions, provided that modelling details are duly considered.

For the application in practice, the three-term classical equation using appropriate reduction factors has the advantage of easy incorporation of foundation geometry, lateral surcharge, and cohesive-frictional soil.

6 REFERENCES

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